KINEMATICS

1. A man is going up in a lift (open at the top) moving with a constant velocity $3\ m/s$. He throws a ball up at $5\ m/sec$ relative to the lift when the lift is 50 m above the ground. Height of the lift when the ball meets it during its downward journey is ($g = 10\ m/s^2$) **[NSEP-2017]**

(A) 53 m (B) 58 m (C) 51 m (D) 68 m

Ans. (A)

Sol. $T = \frac{2(5)}{10} = 1 sec$ $h = 50 + 3(1) = 53\ m$

2. A particle moves according to the law $x = at, y = (\ 1 - \alpha t\ )$ where a and α are positive constants and t is time. The time instant at which velocity makes and angle $\frac{\pi}{4}$ with acceleration. **[NSEP-2018]**

(A) $\frac{4}{\alpha}$ (B) $\frac{3}{\alpha}$ (C) $\frac{2}{\alpha}$ (D) $\frac{1}{\alpha}$

Ans. (D)

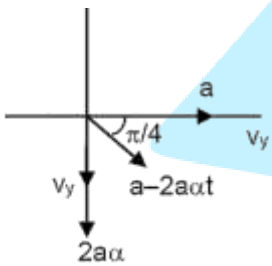
Sol. $x = at$

$v_y = a\ \ a_y = 0$

$y = at - at - a\alpha t^2$

$v_y = a - 2a\alpha t$

$a_y = -2a\alpha\ \ \tan\theta = 1$



$$\frac{a}{-(a - 2a\alpha t)} = 1$$

$a = -a + 2a\alpha t\ \ 2a = 2a\alpha t\ \ t = \frac{1}{\alpha}$

3. The velocity of a projectile at the highest point of its trajectory is $\sqrt{0.4}$ of its velocity at a point at half its maximum height. The angle of projection is. **[NSEP-2018]**

(A) 30° (B) 45° (C) 60° (D) $\tan^{-1}\left(\sqrt{0.4}\right)$

Ans. (C)

Sol. $u_x = \sqrt{0.4}\sqrt{u_y^2 - 2g\frac{1}{2}\frac{u_y^2}{2g} + u_x^2}$

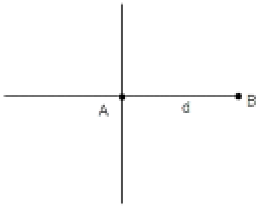
$\Rightarrow \frac{u_y}{u_x} = \sqrt{3} = \tan\theta \Rightarrow \theta = \frac{\pi}{3}$

4. An aircraft flies at a speed v from city A to city B and back in time t_0. City B is to the east of city A at a distance d. The aircraft takes time t_1 for the round trip if wind blows with speed w along AB and time t_2 if the wind blows with the same speed perpendicular to AB. Then, **[NSEP-2018]**

(A) $t_1 = t_2 = t_0$ (B) $t_1 > t_2 > t_0$ (C) $t_1 < t_2 < t_0$ (D) $t_1 > t_0 > t_2$

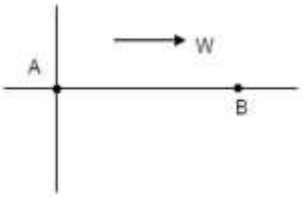
Ans. (B)

Sol. Case – 1



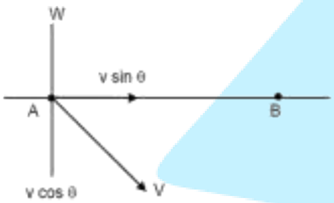
$t_0 = \frac{2d}{v}$

Case – 2



$$t_1 = \frac{d}{v+w} + \frac{d}{v-w} = \frac{2vd}{v^2 - w^2}$$

Case -3



$$t_2 = \frac{2d}{v\sin\theta} = \frac{2d}{\sqrt{v^2 - w^2}}$$

Hence $t_2 > t_0 \Rightarrow t_1 > t_2 > t_0$

5. An observer stands on the platform at the front edge of the first bogie of a stationary train. The train starts moving with uniform acceleration and the first bogie takes 5 seconds to cross the observer. If all the bogies of the train are of equal length and the gap between them is negligible, the time taken by the tenth bogie to cross the observer is **[NSEP-2019]**

(A) 1.07 s (B) 0.98 s (C) 0.91 s (D) 0.81 s

Ans. (D)

Sol. Assume, acceleration of train $= a$

length of each bogie $= L$

Then, from given data $L = \frac{1}{2}a(5)^2$

$$\frac{2L}{25} = a$$

Time taken to cross 9 bogies $\Rightarrow \frac{1}{2}at_1^2 = 9\text{ L}$

$t_1 = \sqrt{\frac{18L}{a}}$

Time taken to cross 10 bogies $\Rightarrow \frac{1}{2}at_2^2 = 10\text{ L}$

$t_2 = \sqrt{\frac{20L}{a}}$

Time taken to cross 10^{th} bogie $= t_2 - t_1 = 0.81\text{sec}$

6. A particle is projected from the ground with a velocity $v = \left(3\hat{i} + 10\hat{j}\right)ms^{-1}$. The maximum height attained and the range of the particle are respectively given by (use $g = 10\ m/s^2$) **[NSEP-2019]**

(A) 5 m and 6 m (B) 3 m and 10 m (C) 6 m and 5 m (D) 3 m and 5 m

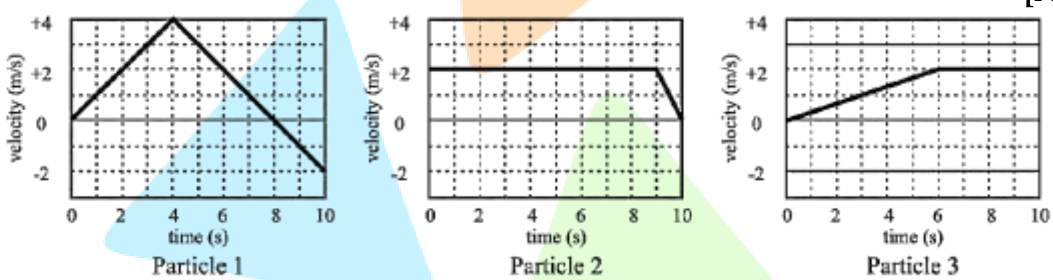
Ans. (A)

Sol. $\vec{V} = 3\hat{i} + 10\hat{j}\ m/s$

Maximum height $= \frac{(10)^2}{2g} = 5\text{m}$

Range $= 2 \cdot \frac{10}{g} \cdot 3 = 6\text{ m}$

7. In the following figures the velocity-time graphs for three particles 1, 2 and 3 are shown. **[NSEP-2019]**



The magnitude of average acceleration of the three particles, over 10 s, bear the relationship.

(A) $a_1 > a_2 > a_3$ (B) $a_2 > a_1 > a_3$ (C) $a_3 > a_2 > a_1$ (D) $a_1 = a_2 = a_3$

Ans. (D)

Sol. Particle

$\frac{\Delta V}{\Delta t} = \frac{-2-0}{10} = \frac{-2}{10} = -0.2 = a_1$

$\frac{\Delta V}{\Delta t} = \frac{0-2}{10} = -0.2 = -0.2 = a_2$

$\frac{\Delta V}{\Delta t} = \frac{2-0}{10} = 0.2 = 0.2 = a_3 \therefore |a_1| = |a_2| = |a_3|$

8. A particle of mass m is thrown vertically up with velocity u. Air exerts an opposing force of a constant magnitude F. The particle returns back to the point of projection with velocity v after attaining maximum height h, then **[NSEP-2019]**

(A) $h = \frac{u^2}{2\left(g + \frac{F}{m}\right)}$ (B) $h = \frac{v^2}{2\left(g - \frac{F}{m}\right)}$ (C) $v = u\sqrt{\frac{\left(g - \frac{F}{m}\right)}{\left(g + \frac{F}{m}\right)}}$ (D) $v = u\sqrt{\frac{\left(g + \frac{F}{m}\right)}{\left(g - \frac{F}{m}\right)}}$

Ans. (ABC)

Sol. During upward motion

$$a=\left(g+\frac{F}{m}\right)\ \therefore h=\frac{u^2}{2a}=\frac{u^2}{2(g+F/m)}$$

During descent

$$a_2=\left(g-\frac{F}{m}\right)\ v=\sqrt{2a_2h}=u\sqrt{\frac{g-\frac{F}{m}}{g+\frac{F}{m}}}\ h=\frac{v^2}{2a_2}=\frac{v^2}{2\left(g-\frac{F}{m}\right)}$$

9. A shot is fired at an angle α to the horizontal up a hill (Considered to be a long straight incline plane) of inclination β to the horizontal. It will strike the hill horizontally if **[NSEP-2020]**

(A) $\tan\alpha=2\tan\beta$ (B) $\sin\alpha=\sin2\beta$ (C) $\sin\alpha=2\sin\beta$ (D) $\tan\alpha=4\tan\beta$

Ans. (A)

Sol. For a projectile the maximum height is $H=\frac{u^2\sin^2\alpha}{2g}$

and the range is $R=\frac{u^2\sin2\alpha}{g}$,

for the given problem

$$\frac{H}{R/2}=\tan\beta\Rightarrow\frac{\sin^2\alpha}{\sin2\alpha}=\tan\beta\Rightarrow\tan\alpha=2\tan\beta$$

10. A particle moves along a straight line. Its displacement S varies with time t according to the law $s^2=at^2+2bt+c$ (a,b and c are constants). The acceleration of this particle varies as **[NSEP-2022]**

(A) S^0 (B) S^{-1} (C) S^{-2} (D) S^{-3}

Ans. (D)

Sol. $s^2=at^2+2bt+c$

Differentiate w.r.t. time, we get

$$2\ S\frac{dS}{dt}=2at+2\ b\ \ s\frac{dS}{dt}=at+b\Rightarrow\frac{dS}{dt}=\frac{at\ b}{S}\ \left(\frac{dS}{dt}=v\right)$$

Again, differentiate w.r.t time, we get

$$\frac{dS}{dt}\left(\frac{dS}{dt}\right)+S\left(\frac{d^2S}{dt^2}\right)=a$$

$$S\left(\frac{d^2S}{dt^2}\right)=a-\left(\frac{dS}{dt}\right)^2$$

$$S\left(\frac{d^2S}{dt^2}\right)=a-\left(\frac{at+b}{S}\right)^2$$

$$S\left(\frac{d^2S}{dt^2}\right)=\frac{aS^2-(at+b)^2}{S^2}$$

$$S\left(\frac{d^2S}{dt^2}\right)=\frac{a\left(at^2+2bt+c\right)-\left(a^2t^2+b^2+2abt\right)}{s^2}$$

$$\frac{d^2s}{dt^2}=\frac{ac-b^2}{s^3}\ \text{i. e. Acc.}\propto S^{-3}$$

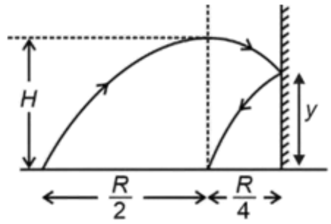
11 A ball is projected from horizontal ground. It attains a maximum height H on its projectile path and there after strikes a stationary smooth vertical wall and falls on ground vertically below the point of maximum height. Assume the collision with wall to be perfectly elastic, the height of the point on the wall where the ball strikes is **[NSEP-2022]**

(A) $3H/4$ (B) $2H/3$ (C) $H/2$ (D) $4H/5$

Ans. (A)

Sol. $\Rightarrow$ Horizontal speed remains same throughout the motion, so the total distance travelled in horizontal

$\Rightarrow x\frac{3R}{4}, \tan\theta = \frac{4H}{R}$



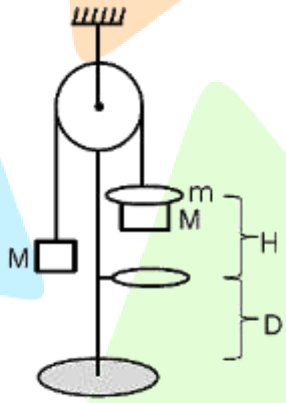
$\rightarrow$ Trajectory

$Py = x\tan\theta\left(1-\frac{x}{R}\right)$

$\rightarrow y = \frac{3R}{4}\times\frac{4H}{R}\left(1-\frac{3}{4}\right)$

$\rightarrow y = \frac{3H}{4}$

12. Two equal blocks, each of mass M, hang on either side of a frictionless light pulley with a light string. A rider of mass m is placed on one of the blocks (as shown). When the system is released, the block with rider descends a distance H till the rider is caught by a ring that allows the block to pass through. The system moves a further distance D taking time t. In such a situation, the acceleration due to gravity is **[NSEP-2023]**



(A) $g = \frac{(2M+m)D^2}{2mHt^2}$ (B) $g = \frac{(M+m)D^2}{2mHt^2}$ (C) $g = \frac{(2M+m)D}{2mHt^2}$ (D) $g = \frac{(M+2m)D}{2mHt^2}$

Ans. (A)

Sol. $a = \frac{(m)g}{2M+m}$

$v = \sqrt{2\cdot\left(\frac{mg}{2M+m}\right)\cdot H}$

$D = \left(\sqrt{\frac{2mg\cdot H}{2M+m}}\right)\cdot t$

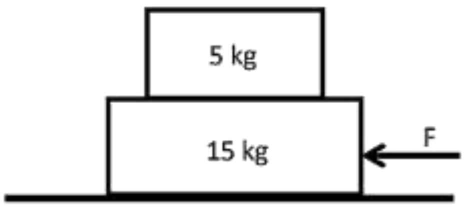
$\frac{D^2}{t^2} = \left(\frac{2mg}{2M+m}\right)H$

$g = \frac{D^2}{t^2}\frac{(2M+m)}{2mH}$

NEWTON'S LAW OF MOTION & FRICTION

1. A constant force F applied to the lower block of mass 15 kg makes it slide between the upper block of mass 5 kg and the table below, as shown. The coefficients of static and kinetic (μ_k) friction between the lower block and the table are 0.5 and 0.4 respectively and those between the two blocks are 0.3 and 0.1. The accelerations of the upper and the lower blocks are respectively.

[NSEP -2018]

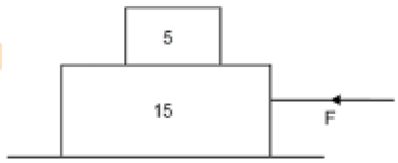


(A) 1.96 m/s^2 and 1.96 m/s^2 (B) 1.96 m/s^2 and 3.92 m/s^2

(C) 0.98 m/s^2 and 0.49 m/s^2 (D) 0.98 m/s^2 and 1.96 m/s^2

Ans. (CD)

Sol. $F - 80 = 20(3) \Rightarrow F = 140$



$a_{upper} = 1 \text{ m/s}^2, a_2 = \dfrac{140 - 80 - 5}{15} = 3.67 \text{ m/s}^2$

When slipping starts acceleration of upper block will be $g/10 = 0.98 \text{ m/s}^2$.

Acceleration of lower block will depend on F so answer may be (C) or (D)

2. A steel cable hanging vertically can support a maximum load W . The cable is cut to exactly half of its original length, the maximum load that it can support now is **[NSEP -2018]**

(A) W (B) $\dfrac{W}{2}$

(C) 2 W (D) More than $\dfrac{W}{2}$ but less than W

Ans. (A)

Sol. Breaking strength remains same $\Rightarrow$ Maximum load remain same

3. A wedge of mass M rests on a horizontal frictionless surface. A block of mass m starts sliding down the rough inclined surface of the wedge to its bottom. During the course of motion, the centre of mass of the block and the wedge system [NSEP -2019]

(A) Does not move at all

(B) Moves horizontally with constant speed

(C) Moves horizontally with increasing speed

(D) Moves vertically with increasing speed

Ans. (D)

Sol. There is no external horizontal force on (block + wedge) system. and vertical external forces are unbalanced therefore D.

4. A freely falling spherical rain drop gathers moisture (maintaining its spherical shape all the way) from the atmosphere at a rate $\frac{dm}{dt}=kt^2$ where t is the time and m is the instantaneous mass of the drop, the constant $k=12gm/s^3$. If the drop, of initial mass $m_0=2gm$, starts falling from rest, the instantaneous velocity of the drop exactly after 5 second shall be (ignore air friction and air buoyancy) **[NSEP -2021]**

(A) $12.4\ ms^{-1}$ (B) $49.0\ ms^{-1}$ (C) $122.5\ ms^{-1}$ (D) data insufficient

Ans. (A)

Sol. According to the Newton's Second Law $F=\frac{dp}{dt}$. In the present case the rain drop is attracted by the earth so at any instant,

$$mg=\frac{d}{dt}(mv)\Rightarrow mg=m\frac{dv}{dt}+v\frac{dm}{dt}\ \text{ or }\ g=\frac{dv}{dt}+\frac{v}{m}\frac{dm}{dt}$$

Given that $\frac{dm}{dt}=kt^2\Rightarrow m=\frac{kt^3}{3}+m_0$

where m_0 is initial mass.

Further $g=\frac{dv}{dt}+\frac{v}{\frac{kt^3}{3}+m_0}\times kt^2\Rightarrow\frac{dv}{dt}=g-\frac{3kt^2}{3m_0+kt^3}\times v\Rightarrow\frac{dv}{dt}+\left(\frac{3kt^2}{3m_0+kt^3}\right)v=g$

or $\Rightarrow\left(3\,m_0+kt^3\right)dv+v^3kt^2dt=g\left(3\,m_0+kt^3\right)dt$ or $\Rightarrow d\left\{\left(3\,m_0+kt^3\right)v\right\}=g\left(3\,m_0+kt^3\right)dt$

Integrating we get

$$\left(3m_0+kt^3\right)v=\left(3m_0gt+\frac{gkt^4}{4}\right)_0^t$$

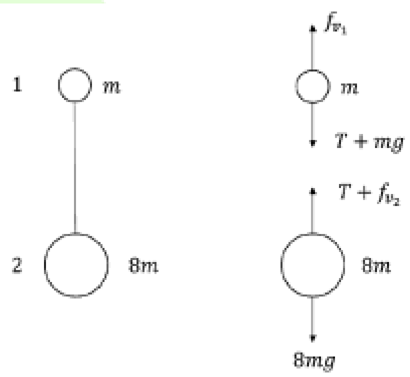
or $v=g\frac{12m_0t+kt^4}{4\left(3m_0+kt^3\right)}=\frac{1905}{1506}g=12.4\ ms^{-1}$

5. Two small solid balls of masses m and $8m$ made up of same material are tied at the two ends of a thin weightless thread. They are dropped from a ballon in air. The tension T of thread during fall, after the motion of balls has reached steady state is **[NSEP -2022]**

(A) 2 mg (B) 3.5 mg (C) 4.5 mg (D) Zero

Ans. (A)

Sol. Viscous force, $F_v=6\pi\eta rv$ … (i)



For both solid balls, $\rho=\frac{8m}{\frac{4}{3}\pi r_2^3}=\frac{m}{\frac{4}{3}\pi r_1^3}$

$\therefore r_2=2r_1$ … (ii)

Motion of balls has reached steady state:

For equilibrium of the ball of mass 8 m ,

$T+F_{v_2}=8mg$ … (iii)

Also, from eqn. (1) & (2) $\Rightarrow F_{v_2}=2F_{v_1}$

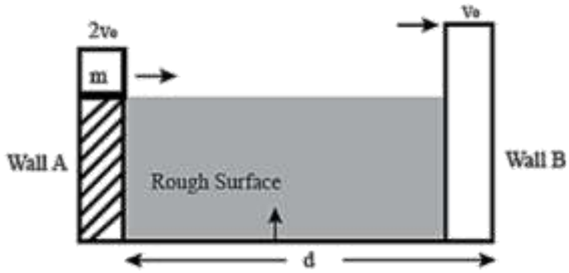
$\therefore$ from eqn. (3) $\Rightarrow T+2F_{v_1}=8mg$

$\Rightarrow T+2(T+mg)=8mg$

$\Rightarrow 3T=6mg$

$\Rightarrow T=2mg\rightarrow$ Option (A).

6. As shown in figure, a block of mass m is projected from wall A with velocity $2v_0$, on the rough surface with constant sliding friction to hit the wall B with velocity v_0. With what velocity same mass m should be projected to hit the wall B with same velocity v_0 if the surface is now moving upwards with an acceleration of $a = 4\,g$ **[NSEP -2022]**



(A) $2v_0$ (B) $3v_0$ (C) $4v_0$ (D) $5v_0$

Ans. (C)

Sol. Case – 1

$v_0^2 = (2v_0) - 2a_1 d \Rightarrow a_1 = \frac{3v_0^2}{2d} = \mu g$ Here $a_1 = \frac{\mu N}{m} = \mu g \{N = mg\}$

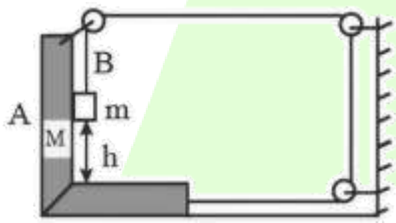
Case – 2

Now $N = m(g + a) = 5mg \Rightarrow a_2 = \frac{\mu N}{m} = 5\mu g$

$a_2 = \frac{15v_0^2}{2d}$ Let $u =$ initial velocity along the surface

$v_0^2 = u^2 - 2a_2\ d\ v_0^2 = u^2 - 2\left(\frac{15v_0^2}{2d}\right)d \Rightarrow v_0^2 = u^2 - 15v_0^2 \Rightarrow u^2 = 16v_0^2 \Rightarrow u = 4v_0$

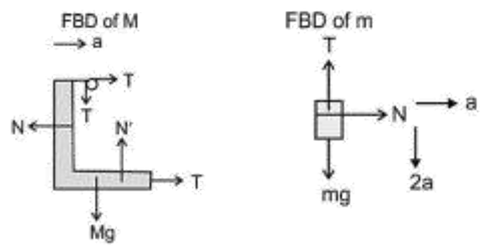
7. A simplification of a kind of interlock is shown in figure. All surfaces are smooth and frictionless. The body m has a mass $m = 1$ kg and the block $M = 15$ kg. The time ' m ' takes to reach the base if it is released at height $h = 4$ meter above the base of M, is [use $g = 10\ \text{ms}^{-2}$] **[NSEP -2022]**



(A) 1 s (B) $\sqrt{3}$ s (C) 2 s (D) $2\sqrt{2}$ s

Ans. (C)

Sol. From the FBDs we can write the equations as,



$2\,T - N = Ma$ … (i)

$N = ma$ … (ii)

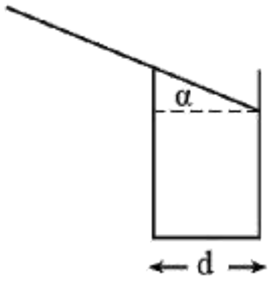
$mg - T = 2ma$ … (iii)

on solving, $2mg = (M + 5\,m)a \Rightarrow a = 1\ \text{ms}^{-2}$

So, the acceleration of block in downward motion is

$2a = 2\ \text{ms}^{-2}$, so time taken is $t = 2$ s

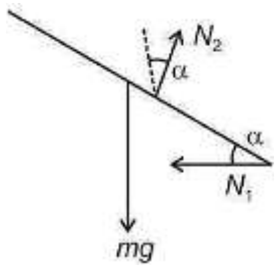
8. A cylindrical tumbler of diameter d has smooth sides and edge. A thin rod of length L is balanced on the edge of the tumbler as shown in figure. The angle α that the rod makes with horizontal for this trick to work is **[NSEP -2022]**



(A) $\sin^{-1}\left(\frac{d}{L}\right)^{\frac{1}{2}}$ (B) $\cos^{-1}\left(\frac{2d}{L}\right)^{\frac{1}{3}}$ (C) $\cos^{-1}\left(\frac{d}{L}\right)^{\frac{1}{3}}$ (D) $\sin^{-1}\left(\frac{2d}{L}\right)^{\frac{1}{2}}$

Ans. (B)

Sol. Writing the equation of NLM,



$N_2\cos\alpha = mg$ ... (i)

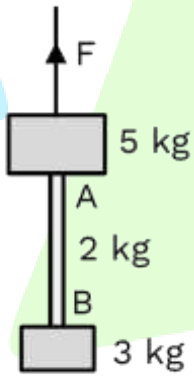
Taking torque from point of application of N_1,

$N_2\ d\sec\alpha = mg\frac{L}{2}\cos\alpha$ ... (ii)

On solving $\alpha = \frac{2d}{L}$

$\alpha = \cos^{-1}\left(\frac{2d}{L}\right)^{\frac{1}{3}}$

9. A solid block of mass 3 kg is suspended from the bottom of a 5 kg block with the help of a rope AB of mass 2 kg as shown in the figure. When pulled by an upward force F, the whole system experiences an upward acceleration $a = 2.19\ ms^{-2}$. Choose the correct option **[NSEP -2022]**



(A) Net force on the rope AB is 24 N

(B) Tension at the midpoint of the rope AB is 48 N

(C) Force F is 20 N

(D) Force F is 60 N

Ans. (B)

Sol. Since acceleration is $2.19\ m/s^2$

$F - 10\times 9.81 = 10\times 2.19\left[\ g = 9.81\ m/s^2\right] F = 21.9 + 98.1$

$T = 220\ N$ Net force on the rope $F_R = m_R a_R = 2\times 2.19 = 4.38\ N$

Let tension at midpoint is T, then

$T - 4\times g = 4\times a\ \ T = 4\times(9.81\times 2.19) = 48\ N$

WORK, POWER, ENERGY

1. A body of mass 1.0 kg moves in $X-Y$ plane under the influence of a conservative force. Its potential energy is given by $U = 2x + 3y$ where (x, y) denote the coordinates of the body. The body is at rest at $(2, -4)$ initially. All the quantities have SI units. Therefore, the body

[NSEP-2017]

(A) Moves along a parabolic path
(B) Moves with a constant acceleration
(C) Never crosses the X axis
(D) Has A speed of $2\sqrt{13}\ m/s$ at time $t = 2\ s$

Ans. (BCD)

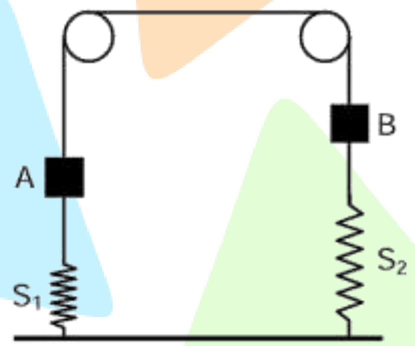
Sol. $F = -2\hat{i} - 3\hat{j}$

$\bar{a} = -2\hat{i} - 3\hat{j}$

(B) correct

(D) $\bar{v} = 2\sqrt{4+9} = 2\sqrt{13}$

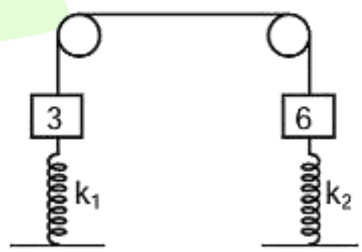
2. In the figure shown below masses of blocks A and B are 3 kg and 6 kg respectively. The force constants of springs S_1 and S_2 are $160\ N/m$ and $40\ N/m$ respectively. Length of the light string connecting the blocks is $8\ m$. The system is released from rest with the springs at their natural lengths. The maximum elongation of spring S_1 will be : **[NSEP -2017]**



(A) 0.294 m
(B) 0.490 m
(C) 0.588 m
(D) 0.882 m

Ans. (A)

Sol. $6g(x) - 3g(x) = \frac{1}{2}k_2x^2 + \frac{1}{2}k_1x^2$



$6g = (k_1 + k_2)x$

$= \frac{6 \times 9.8}{200} = \frac{3 \times 9.8}{100}$

$u^2 = 0.294\ m$

3. A body of mass 4 kg moves under the action of a force $\vec{F} = (4\hat{i} + 12t^2\hat{j})N$, where t is the time in second. The initial velocity of the particle is $(2\hat{i} + \hat{j} + 2k)ms^{-1}$. If the force is applied for 1 s, work done is : **[NSEP-2017]**

(A) 4 J
(B) 8 J
(C) 12 J
(D) 16 J

Ans. (D)

Sol. $\int F dt = \Delta\vec{p}$

$\int_0^1 \left(4\hat{i} + 12\hat{t}\right) dt = m\left(\vec{v}_f - \vec{v}_i\right)$

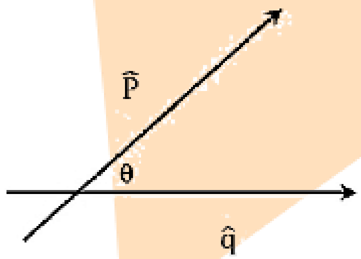
$\Rightarrow 4\hat{i} + \frac{12}{3} t^3 \Big|_0^1 \hat{j} = 4\left(\vec{v}_f - \vec{v}_i\right) \Rightarrow \hat{i} + \hat{j} = \vec{v}_f - \left(2\hat{i} + \hat{j} + 2k\right)$

$\Rightarrow \bar{v}_f = 3\hat{i} + 2\hat{j} + 2k \Rightarrow \left|\bar{V}_f\right| = \sqrt{9+4+4} = \sqrt{17}$

$\left|\bar{V}_i\right| = \sqrt{4+1+4} = 3$

$\Delta K.E = \frac{1}{2} 4(17-9) = 2 \times 8 = 16$ J

4. Motion of a particle in a plane is described by the non-orthogonal set of coordinates (p, q) with unit vectors (p, q) inclined at an angle θ as shown in the diagram. If the mass of the particle is m, its kinetic energy is given by ($\dot{x} = \frac{dx}{dt}$) **[NSEP-2017]**



(A) $\frac{1}{2} m\left(\dot{p}^2 + \dot{q}^2 + \dot{p}\dot{q}\cos\theta\right)$ (B) $\frac{1}{2} m\left(\dot{p}^2 + \dot{q}^2 - \dot{p}\dot{q}(1-\sin\theta)\right)$

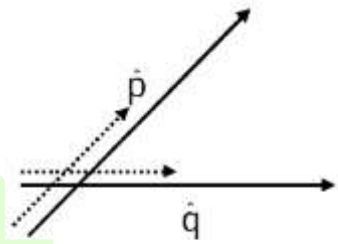
(C) $\frac{1}{2} m\left(\dot{p}^2 + \dot{q}^2 + 2\dot{p}\dot{q}\cos\theta\right)$ (D) $\frac{1}{2} m\left(\dot{p}^2 + \dot{q}^2 + p\ddot{q}\cot\theta\right)$

Ans. (C)

Sol. $r = \left(pp + qq\right)$ $\vec{v} = \frac{dr}{dt} = \frac{dp}{dt} p + \frac{dq}{dt} q$

$\vec{v} \cdot \vec{v} = \left(\frac{dp}{dt}\right)^2 + \left(\frac{dq}{dt}\right)^2 + 2pq\cos\theta$

K.E. $= \frac{1}{2} m\left(p^2 + q^2 + 2pq\cos\theta\right)$



5. The potential energy of a particle of mass m in conservative force field can be expressed as $U = \alpha x - \beta y$ where (x, y) denote the position coordinates of the body. The acceleration of the body is: **[NSEP-2018]**

(A) $\frac{\alpha - \beta}{m}$ (B) $\frac{\alpha + \beta}{m}$ (C) $\frac{\sqrt{\alpha^2 - \beta^2}}{m}$ (D) $\frac{\sqrt{\alpha^2 + \beta^2}}{m}$

Ans. (D)

Sol. $U = ax - by$

$F_x = -a$ $F_y = b$

$F_{net} = \sqrt{a^2 + b^2}$ $a = \frac{\sqrt{a^2 + b^2}}{m}$

6. The resistive force on an airplane flying in a horizontal plane is given by $F_f = kv^2$, where k is constant and v is the speed of the airplane. When the power output from the engine is P_0, the plane flies at a speed v_0. If the power output of the engine is doubled the airplane will fly at a speed of **[NSEP-2019]**

(A) $1.12v_0$ (B) $1.26v_0$ (C) $1.41v_0$ (D) $2.82v_0$

Ans. (B)

Sol. Power = F.V.

$P_0 = KV_0^2 \cdot V_0 = KV_0^3 \#(1)$

If power is doubled

$2P_0 = kV^3 \#(2)$

From equation (1) & (2)

$$\frac{KV^3}{KV_0^3} = \frac{2P_0}{P_0}$$

$V = 2^{1/3}\ V_0 = 1.26\ V_0$

7. If the force acting on a body is inversely proportional to its speed, the kinetic energy of the body varies with time t as [NSEP -2019]

(A) t^0 (B) t^1 (C) t^2 (D) t^{-1}

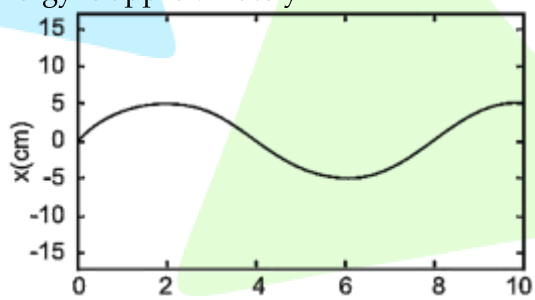
Ans. (B)

Sol. $F \propto \frac{1}{V}$

$F = \frac{k}{V}$ mv dv = kdt Integrating both side

$\frac{1}{2}mv^2 = kt\ \frac{1}{2}mv^2 \propto t$

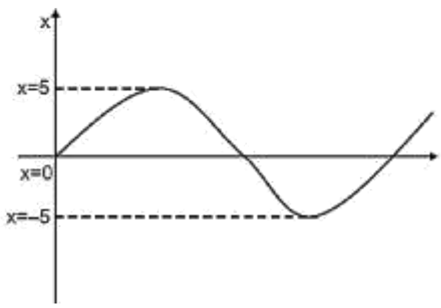
8. Position-time graph of a particle moving in a potential field is shown beside. If the mass of the particle is 1 kg its total energy is approximately **[NSEP-2019]**



(A) 15.45×10^{-4} J (B) 30.78×10^{-4} J (C) 7.71×10^{-4} J (D) 3.85×10^{-4} J

Ans. (C)

Sol. $A = 5$ m



$T = 8$sec

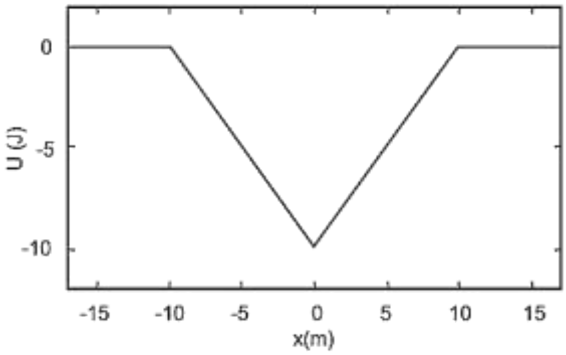
$$\omega = \frac{2\pi}{8} = \frac{\pi}{4}$$

$k = m\omega^2$

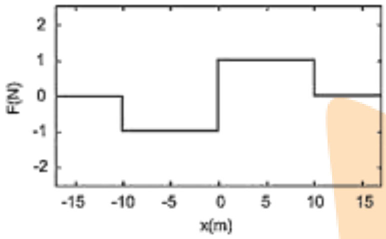
$$k = 1\times\frac{\pi^2}{16} = \frac{\pi^2}{16}$$

$$TE = \frac{1}{2}kA^2 = \frac{25\pi^2}{32\times100\times100} = 7.71\times10^{-4}\ J$$

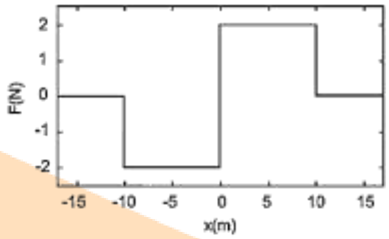
9. The potential energy (U) of a particle moving in a potential field varies with its displacement (x) as shown below. **[NSEP-2019]**



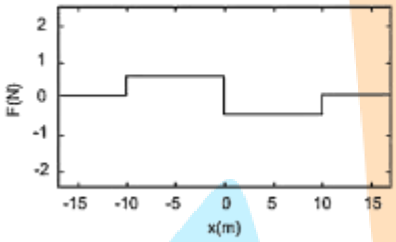
The variation of force $F(x)$ acting on the particle as a function of x can be represented by



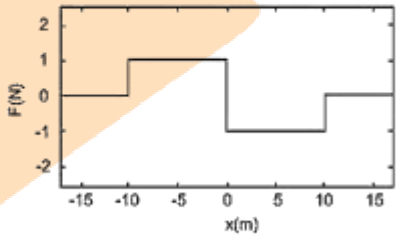
(Fig i)



(Fig ii)



(Fig iii)



(Fig iv)

(A) Fig (i) (B) Fig (ii) (C) Fig (iii) (D) Fig (iv)

Ans. (D)

Sol. for $-20 \le x \le -10$ $-10 \le x \le 0$

$0 \le x \le 10$

$10 < x \le 20$

$U = 0$

$\frac{du}{dx} = -1$

$\frac{du}{dx} = 1$

$\frac{du}{dx} = 0$

$\therefore F = \frac{-du}{dx} = 0$

$F = +1$

$F = -1$

$\therefore F = 0$

10. The kinetic energy of a particle moving along a circle of radius R depends upon the distance covered ' s ' as $KE = as^2$ where a is a constant. The magnitude of the force acting on the particle as a function of ' s ' is **[NSEP-2020]**

(A) $\frac{2as^2}{R}$ (B) $\frac{2as^2}{m}$ (C) 2 as (D) $2as\sqrt{1+\left(\frac{s}{R}\right)^2}$

Ans. (D)

Sol. The velocity is changing on circular path so centripetal acceleration and tangential acceleration both Further

Given is that $\frac{1}{2}mv^2 = as^2 \Rightarrow \frac{mv^2}{R} = \frac{2as^2}{R}$ centripetal force

Also $mv^2 = 2as^2 \Rightarrow v = \sqrt{\frac{2a}{m}}s \Rightarrow \frac{dv}{dt} = \sqrt{\frac{2a}{m}}\frac{ds}{dt} = \sqrt{\frac{2a}{m}}\sqrt{\frac{2a}{m}}s$

$\Rightarrow \frac{dv}{dt} = \frac{2a}{m}s \Rightarrow m\frac{dv}{dt} = 2as =$ tangential force

Net force as a function of s is $F = \sqrt{F_T^2 + F_R^2} \Rightarrow 2as\sqrt{1+\left(\frac{s}{R}\right)^2}$

11. A motor pump is used to deliver water at a certain rate r from a given pipe. To obtain thrice as much water from the same pipe in the same time, the power of the motor has to be increased to **[NSEP-2022]**

(A) 3 times (B) 9 times (C) 27 times (D) 81 times

Ans. (C)

Sol. Mass flow rate, $\frac{dm}{dt} = \rho AV$

According to problem the new flow rate, $\left(\frac{dm}{dt}\right)' = 3\times\left(\frac{dm}{dt}\right)$

$A'v'\rho' = 3\times\rho Av$ ($A' = A \rightarrow$ Same pipe, $\rho = \rho \rightarrow$ Same liquid)

$\Rightarrow v' = 3v$ Also

Force $\Rightarrow \frac{F'}{F} = \frac{v\left(\frac{dm}{dt}\right)}{v\left(\frac{dm}{dt}\right)} = \frac{3v\times3\left(\frac{dm}{dt}\right)}{v\times\frac{dm}{dt}}$

$\Rightarrow F' = 9F \therefore$ Power $\Rightarrow \frac{P'}{P} = \frac{F'v'}{Fv} = \frac{9F\times3v}{Fv} = 27$

$\therefore P' = 27P$ i.e., 27 times $\rightarrow$ option C.

12. A particle of mass m moves in a straight line under the influence of a certain force such that the power (P) delivered to it remains constant. Starting from rest, the straight-line distance traveled by the moving particle in time t is **[NSEP-2023]**

(A) $\left(\frac{8Pt^3}{27\,m}\right)^{\frac{1}{2}}$ (B) $\left(\frac{4Pt^3}{27\,m}\right)^{\frac{1}{2}}$ (C) $\left(\frac{8Pt^2}{9\,m}\right)^{\frac{1}{2}}$ (D) $\left(\frac{8Pt^3}{9m}\right)^{\frac{1}{2}}$

Ans. (D)

Sol. Since, power, $P =$ constant

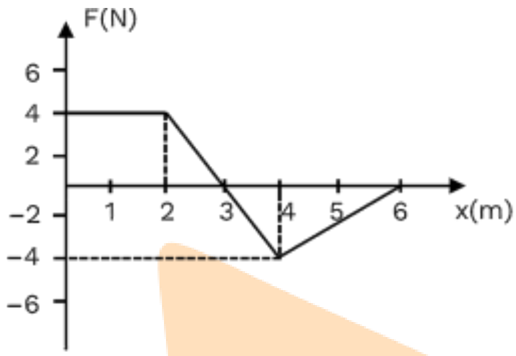
$P = F.V$. since particle is moving in straight line.

$\therefore mv\frac{dv}{dt} = P$ or $m\int_0^v vdv = \int Pdt \Rightarrow \frac{v^2}{2} = \frac{Pt}{m} \Rightarrow v = \sqrt{\frac{2Pt}{m}}$

$$\frac{ds}{dt}=\sqrt{\frac{2Pt}{m}} \Rightarrow \int_0^s ds=\int_0^t\sqrt{\frac{2Pt}{m}}dt$$

$$S=\sqrt{\frac{2P}{m}}\times\frac{t^{3/2}}{3/2}=\sqrt{\frac{8Pt^3}{9m}}$$

13. The force $F(x)$ acting on a body of mass m changes with position x (in meter) as shown. It is given that the potential energy $U(x)=0$ at $x=0$ Choose correct option(s). **[NSEP-2023]**



(A) $U(x)=0$ at $x=0, x=3$ and $x=6$

(B) $U(x)=2x^2-12x$ for $2\le x\le 4$

(C) $U(x)=-x^2+12x-40$ for $4\le x\le 6$

(D) At $x=3, U(x)=-10J$

Ans. (CD)

Sol. In interval $(0,2)$

$F = 4$

$U=-4x$ u at $x=2=-8$

In interval $(2,4)$

$F=-4x+12$

$$\int_{-8}^{u}\partial U=\int_2^x F\partial x=\int_2^x(-4x+12)dx=\left[2x^2-12x\right]_2^x$$

$U+8=2x^2-12x-8+24$

$U=2x^2-12x+8$

u at $x=3$

$U=18-36+8=-10J$

$U=$ at $x=4$

$U=32-48+8=-8J$

For interval $(4,6)$

$$F=\frac{4}{2}x-12$$

$F=2x-12$

$$\int_{-8}^{U}\partial U=-\int_4^x F\partial x$$

$$U+8=\left[x^2-12x\right]_4^x=-\left[x^2-12x-16+48\right]$$

$U+8=-x^2+12x-32$

$U=-x^2+12x-40$

Correctio option (CD).

4. A home aquarium partly filled with water slides down an inclined plane of inclination angle θ with respect to the horizontal. The surface of water in the aquarium [NSEP -2019]

(A) remains horizontal

(B) remains parallel to the plane of the incline.

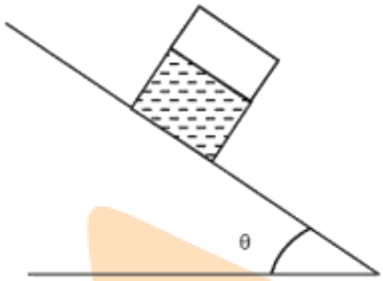
(C) forms an angle α with the horizon where $0 < \alpha < \theta$

(D) forms an angle α with the horizon where $0 < \alpha < 90$

Ans. (B or C)

Sol. if $\mu = 0$

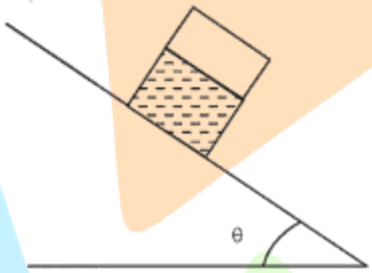
$\alpha = \theta$



Therefore if the surface is assume to be smooth only then correct answer is B.

if $\mu = \mu s$

$\alpha = 0$



So $0 \leq \propto < \theta$

Therefore correct option is (C)